Prove that the composite of two reflections over two parallel lines produces a translation that is double the distance between the lines and in the direction of the mapping of the first line to the second line.

Case one – If the point is between the two parallel lines.

Proof:
1. Construct a line perpendicular to lines 1 and 2. Since the lines are parallel a line perpendicular to one of the lines will be perpendicular to the other parallel line. Label the intersection with line 1 as point B and the intersection with line 2 as point C. Label the distance from A to B as x; AB = x.
Find A' after A has been reflected over Line 1. Mark A'B = x because reflection maintains distance.

Reflect A' over line 2 and label A''

Since the distance from A to C is unknown, label that distance y; AC = y. Therefore, the distance from C to A'' is 2x + y; A''C = 2x + y.
The distance between the two parallel lines is $BC = x + y$. The distance between $A$ and $A''$ is $AA'' = 2x + 2y$.

Therefore, the distance that $A$ translated because of the composite from $A$ to $A''$ is twice the distance between the two lines and the translation moved right because Line 1 moves right to map to Line 2.

You are to prove the other four cases!
Case two – The point is on the left of the two parallel lines.

Given: $\text{Line 1} \parallel \text{Line 2}$

$r_{\text{Line 2}} \circ r_{\text{Line 1}} \ (\text{point A})$

Case three – The point is on the right of the two parallel lines.

Given: $\text{Line 1} \parallel \text{Line 2}$

$r_{\text{Line 2}} \circ r_{\text{Line 1}} \ (\text{point A})$
Cases four and five – The point is on one of the reflecting lines.